Number State (Fock States)

The total energy of a simple harmonic oscillator is given by -

$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \quad \text{—— (1)}$$

$p =$ momentum $\qquad \omega =$ oscillation freq.

$x =$ Position $\qquad m =$ Mass.

The corresponding operator is called Hamiltonian operator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 \quad ; \quad \hat{p} = -i\hbar \frac{\partial}{\partial x} \quad \text{—— (2)}$$

Here $\hat{x}, \hat{p}$ are two Hermitian operators. Now we define two non-Hermitian operator a and a^{+} such that

$$a = \frac{1}{\sqrt{2m\omega\hbar}} \left(\hat{x} + \frac{i\hat{p}}{m\omega}\right) \quad \text{—— (3a)}$$

and $$a^{+} = \frac{1}{\sqrt{2m\omega\hbar}} \left(\hat{x} - \frac{i\hat{p}}{m\omega}\right) \quad \text{—— (3b)}$$

$\Rightarrow \hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (a + a^{+})$.

$\Rightarrow \hat{p} =$

In terms of this two non hermitian operators a and a^{+}. the Hamiltonian Operator $\hat{H}$ can be written as.

$$\hat{H} = \hbar\omega \left(a^{+}a + \frac{1}{2}\right). \quad \text{—— (4)}$$

Here $a^{+}a$ is called the number operator $\hat{n}$.

Here $\hat{a}$ is called the destruction or annihilation operator as it annihilate one quantum of energy and a^{+} is called creation operator as it does the opposite

$\hat{a}$ and $\hat{a}^{+}$ do not commute.

$$[\hat{a}, \hat{a}^{+}] = 1. \quad \text{—— (5)}$$

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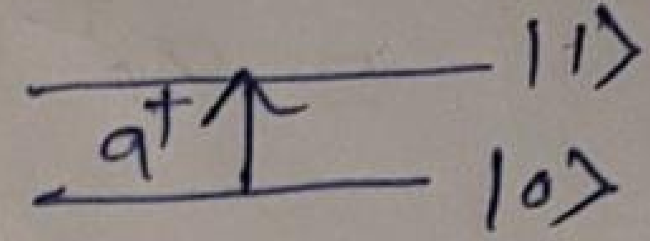
We can construct the higher energy states from the ground state $|0\rangle$ by operating a^{+}.

$$a^{+}|0\rangle = \sqrt{1}\,|1\rangle \qquad \text{as we know } \cdot\ a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$\text{or } |1\rangle = \frac{1}{\sqrt{1}}\, a^{+}|0\rangle$$



$$a^{+}|1\rangle = \sqrt{2}\,|2\rangle$$

$$\Rightarrow |2\rangle = \frac{1}{\sqrt{2}}\, a^{+}|1\rangle = \frac{1}{\sqrt{2}}\, a^{+} \frac{1}{\sqrt{1}}\, a^{+}|0\rangle$$

$$\text{or } |2\rangle = \frac{1}{\sqrt{2\cdot 1}}\,(a^{+})^{2}\,|0\rangle.$$

$$\text{Similarly } |3\rangle = \frac{1}{\sqrt{3\cdot 2\cdot 1}}\,(a^{+})^{3}\,|0\rangle \quad \text{and so on.}$$

hence

$$|n\rangle = \frac{1}{\sqrt{n\cdot(n-1)\cdots 1}}\,(a^{+})^{n}\,|0\rangle$$

$$= \frac{(a^{+})^{n}}{\sqrt{n!}}\,|0\rangle.$$

Now we will calculate the expectation value of ~~$\langle x\rangle$ on~~ x and p in the Number state $|n\rangle$

$$\langle x\rangle = \langle n|x|n\rangle = \langle n|x_0(a+a^{+})|n\rangle$$

$$= x_0\langle n|a+a^{+}|n\rangle = x_0\left(\langle n|a|n\rangle + \langle n|a^{+}|n\rangle\right)$$

$$\langle x\rangle_n = 0$$

Similarly $\langle p\rangle_n = 0$.

Thus the expectation value of x and p in the Number state $|n\rangle$ is zero.

Now to calculate the fluctuation we use the formula

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

$$\langle x^2 \rangle_n = \langle n | x^2 | n \rangle$$

$$= x_0^2 \langle n | (a + a^\dagger)^2 | n \rangle$$

$$= x_0^2 \langle n | a^2 + a^{\dagger 2} + a a^\dagger + a^\dagger a | n \rangle$$

$$= x_0^2 \left[\langle n | a^2 | n \rangle + \langle n | a^{\dagger 2} | n \rangle + \langle n | (1 + 2a^\dagger a) | n \rangle \right]$$

$$= x_0^2 \left[0 + 0 + \langle n | n \rangle + 2 \langle n | a^\dagger a | n \rangle \right]$$

$$= x_0^2 (1 + 2n) = \frac{\hbar}{2m\omega}(2n+1) \; ; \quad x_0 = \sqrt{\frac{\hbar}{2m\omega}}$$ is the zero point oscillator.

$$\langle x^2 \rangle_n = \frac{\hbar}{m\omega}\left(n + \frac{1}{2}\right)$$

and $\langle x \rangle_n = 0$

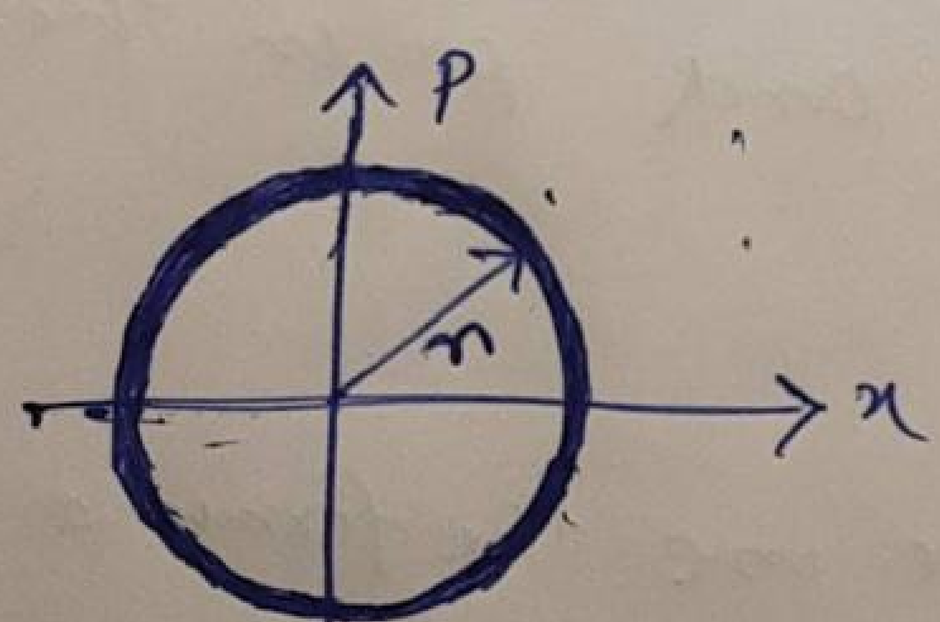
so $\Delta x = \left\{ \langle x^2 \rangle - \langle x \rangle \right\}^{1/2} = \sqrt{\frac{\hbar}{m\omega}} \left(n + \frac{1}{2}\right)^{1/2}$

Similarly for Δp.

$$\Delta p = \left\{ \langle p^2 \rangle - \langle p \rangle \right\}^2$$

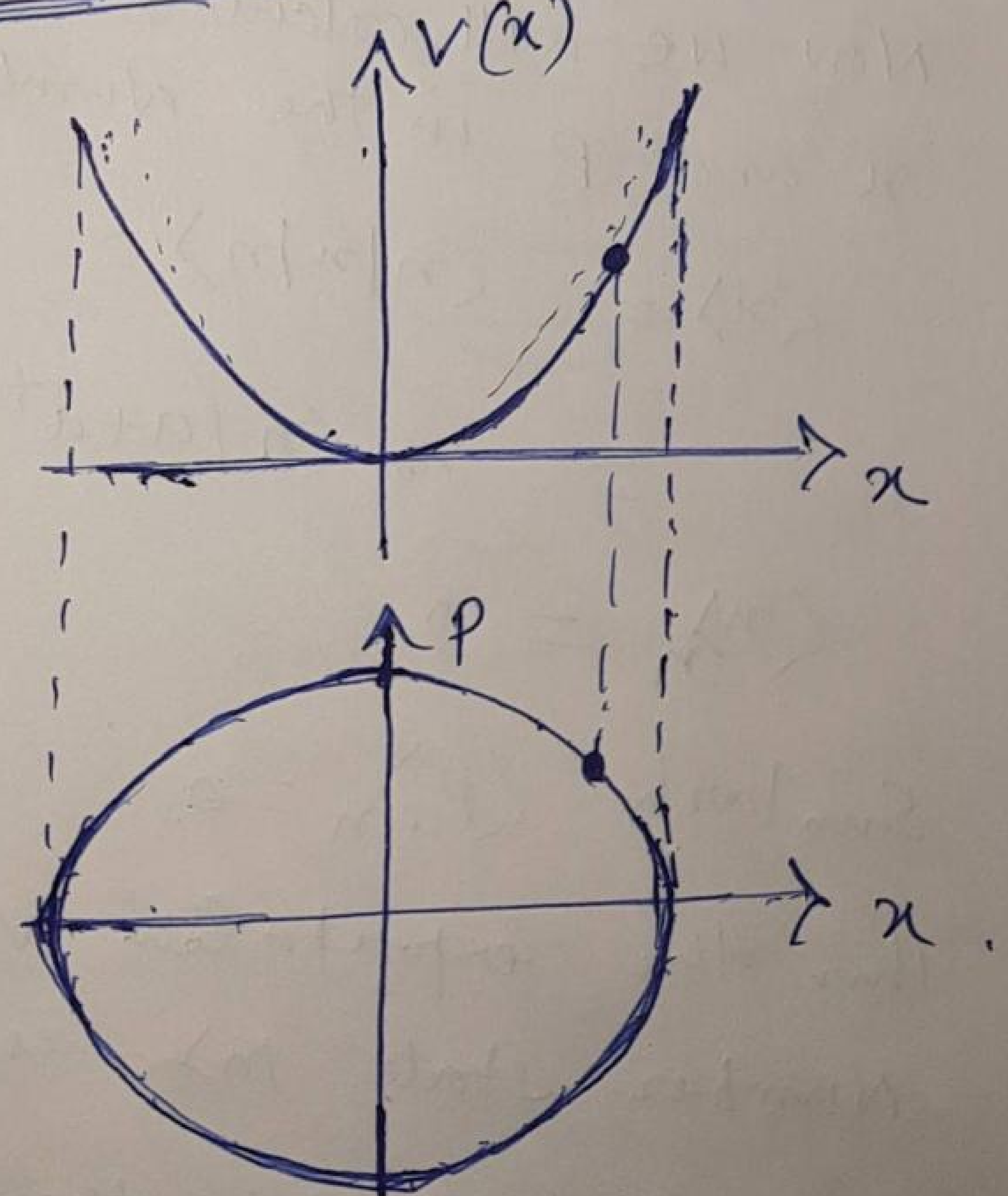
$$=$$

Thus we see that Δx and Δp do not vanish although $\langle x \rangle = \langle p \rangle = 0$



Fock state

x and p has same fluctuations



Phase space representation of classical Harmonic oscillator x, p has a definite value

Time evolution of $\hat{a}$ and $\hat{a}^{+}$.

From Heisenberg equation of motion. we have -

$$\dot{a} = \frac{1}{i\hbar}[a, H] \quad ; \quad H = \hbar\omega a^{+}a.$$

where we neglected the constant zero point oscillation term $\frac{1}{2}\hbar\omega$.

$$\text{or} \quad \dot{a} = \frac{1}{i\hbar}[a, \hbar\omega a^{+}a]$$

$$= \frac{\hbar\omega}{i\hbar}[a, a^{+}a]$$

$$= \frac{\hbar\omega}{i\hbar}[a, a^{+}]a = -i\omega a.$$

$$\frac{da}{dt} = -i\omega a$$

$$\boxed{a(t) = a(0)\, e^{-i\omega t}} \quad ; \quad a(0) = a(t)\big|_{t=0}.$$

Similarly $\quad a^{+}(t) = a^{+}(0)\, e^{i\omega t}$

The number states forms a complete set of orthonormality

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = 1 \quad \text{and} \quad \langle m|n\rangle = \delta_{mn} \; \begin{array}{ll} = 0 & m \neq n \\ = 1 & m = n \end{array}$$

Thus the eigenvectors of the number operators are mutually orthogonal.

The average value of $\hat{a}$ in number state $|n\rangle$ is given by –

$$\langle \hat{a} \rangle = \langle n|\hat{a}|n\rangle = \langle n|n-1\rangle \sqrt{n} = 0$$

similarly $\langle \hat{a}^{\dagger} \rangle = \sqrt{n+1}\, \langle n|n+1\rangle = 0.$

So, ~~But~~ the average value of electric field $E \propto a + a^{\dagger}$ is also 0

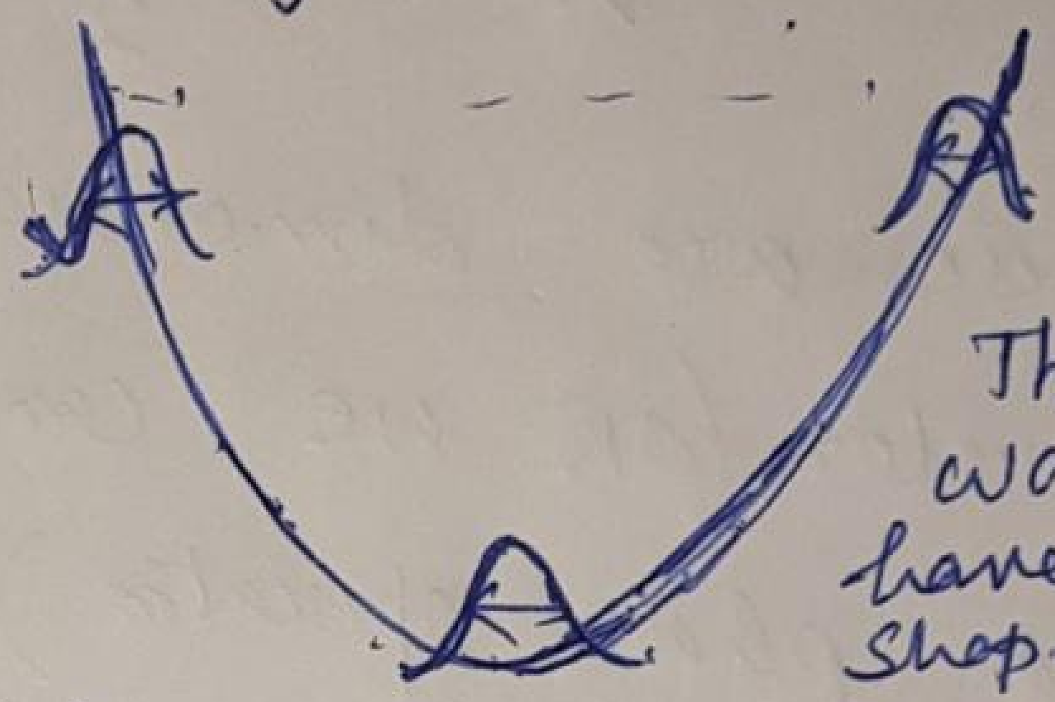
$$\langle \hat{E} \rangle_n = 0.$$

This is not expected classically. Because we have n photons and average electric field vanishes. So Fock states are of purely quantum origin.

Coherent States

The Quantum Mechanical analogue of a classical e.m. wave is called the coherent states. This is a quasiclassical state (normally produced by intense lasers). Coherent states are the eiginstate of the anhilation operator $\hat{a}$.

$$\boxed{\hat{a}|\alpha\rangle = \alpha|\alpha\rangle}$$



The three wave packet have equal shape

Minimum uncertainty wave packet oscillating in a Harmonic oscillator Potential

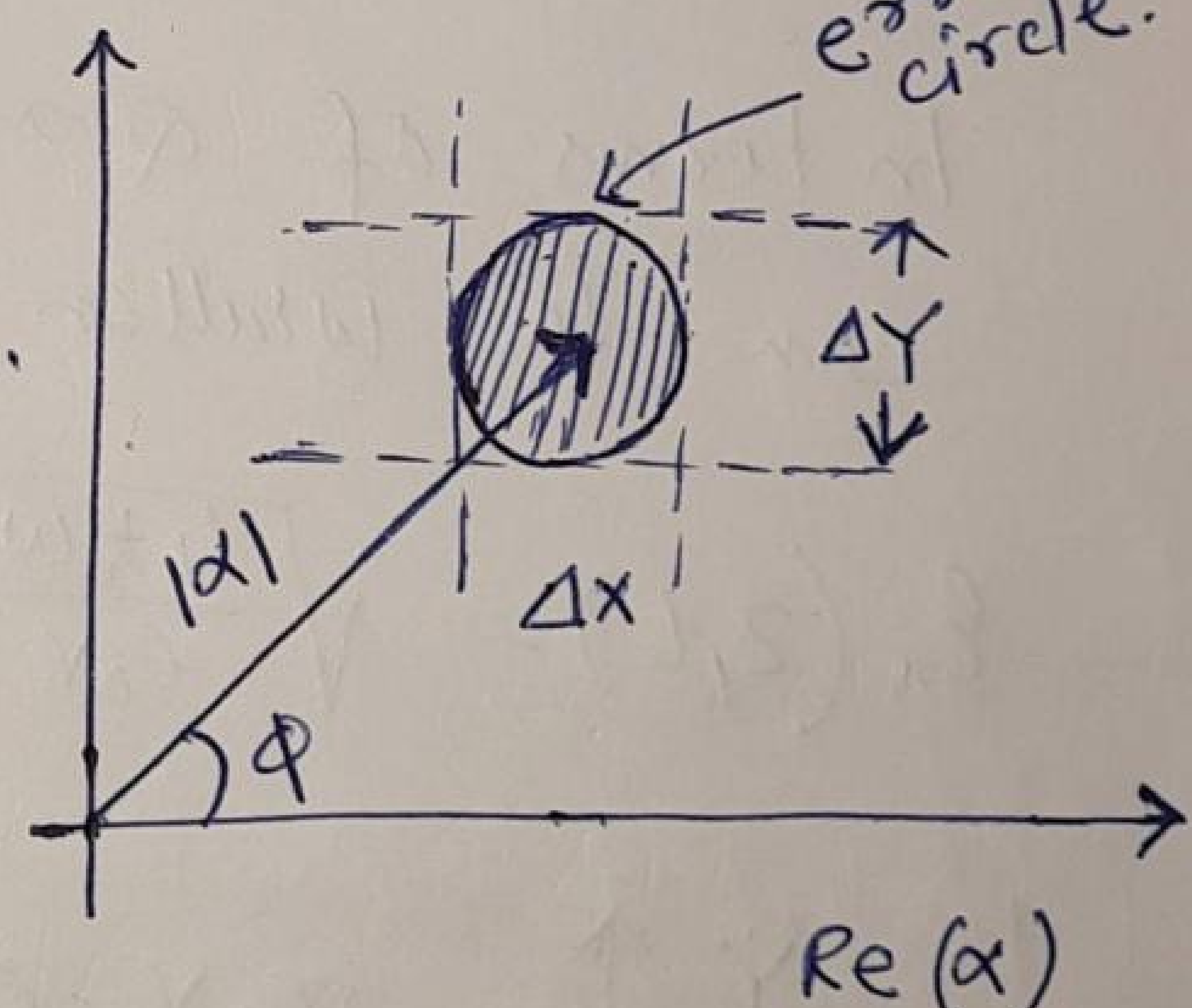
Here $|\alpha\rangle$ is the coherent state.

$\alpha = |\alpha| e^{i\phi}$ is a complex quantity

$|\alpha|$ = Amplitude and ϕ is the phase.

The Complex quantity α can be written as–

$$\boxed{\alpha = X + iY} = Re(\alpha) + i\,Im(\alpha)$$



Phasor diagram of coherent state $|\alpha\rangle$.

Here X and Y are two dimensionless quadratures of the field.

$$|\alpha| = \sqrt{X^2 + Y^2}$$

$$X = |\alpha| \cos\phi$$

$$Y = |\alpha| \sin\phi$$

$$\boxed{\Delta X = \Delta Y} = \frac{1}{2}$$

So they are minimum uncertainty wave packet with equal uncertainties in X and Y.

Classically $|\alpha|$ is related to electric field amplitude.

$$|\alpha| = \sqrt{\frac{\epsilon_0 V}{4\hbar W}}\,\mathcal{E}_0 = \sqrt{\frac{\epsilon_0 V}{4\hbar\omega}}\,\mathcal{E}_0$$

and

$$E_{class} = \hbar\omega |\alpha|^2 .$$

The Quadrature operators corresponding to the Quadrature observables are -

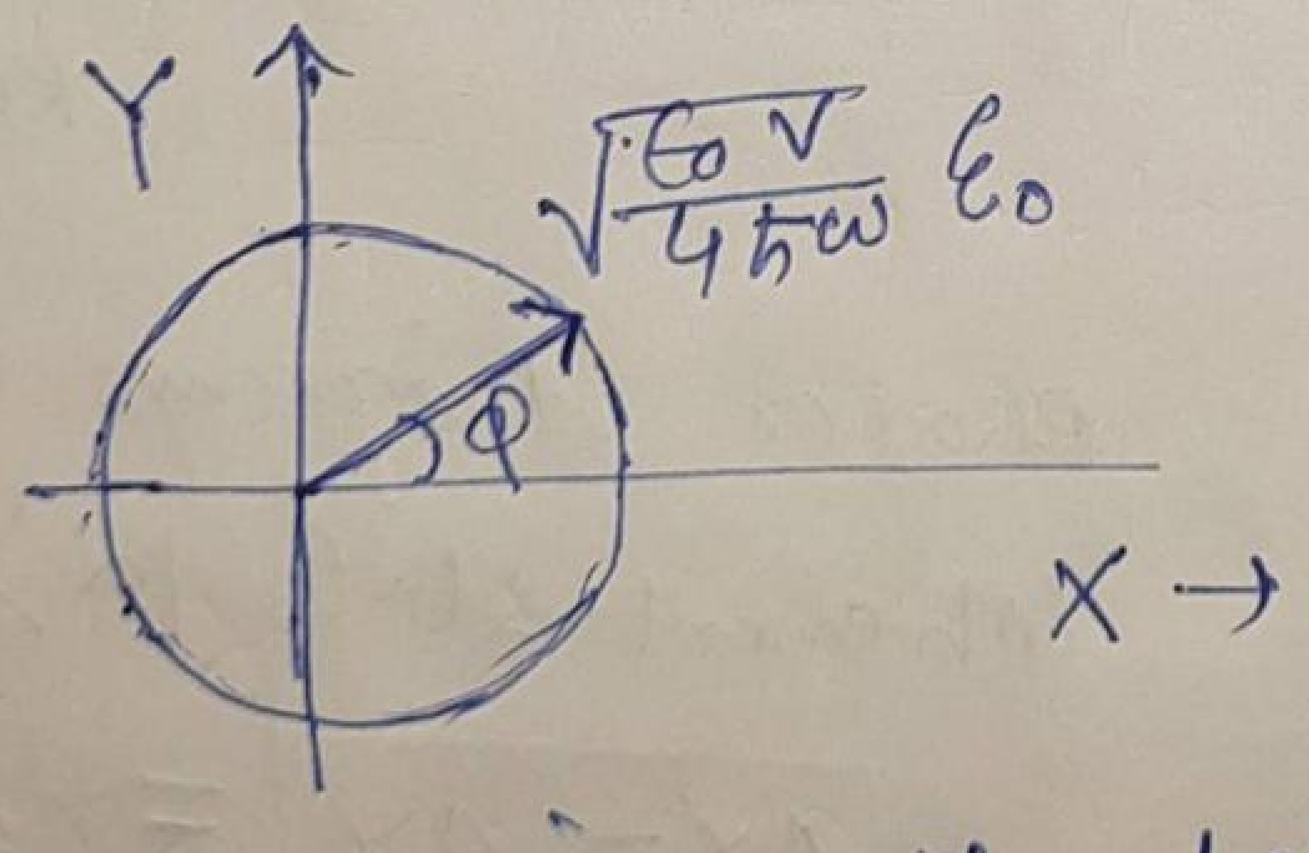
$$\hat{X} = \frac{\hat{a} + a^{+}}{2} \quad \text{and} \quad \hat{Y} = \frac{a - a^{+}}{2i}$$

These are dimensionless position and momentum operator (OR we can call dimensionless amplitude and phase operator).

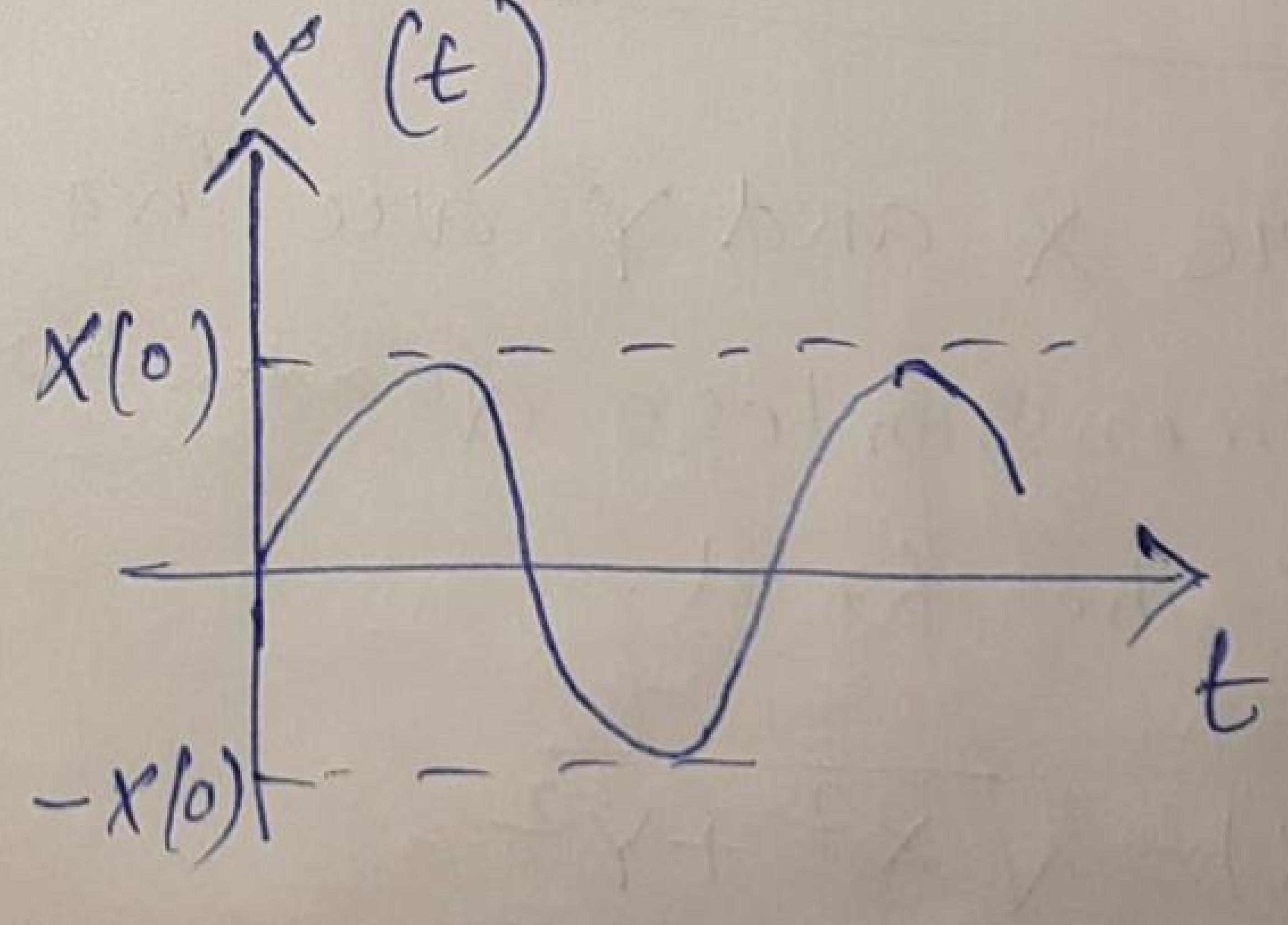
$$\left.\begin{aligned} X(t) &= \left(\frac{\epsilon_0 V}{4\hbar\omega}\right)^{1/2} \mathcal{E}_0 \sin\omega t . \\ Y(t) &= \left(\frac{\epsilon_0 V}{4\hbar\omega}\right)^{1/2} \mathcal{E}_0 \cos\omega t . \end{aligned}\right\}$$

In terms of Quadrature ~~operators~~ the electric field can be written as

$$\mathcal{E}_x(z,t) = \sqrt{\frac{4\hbar\omega}{\epsilon_0 V}} \sin kz \left(\cos\phi \; X(t) + \sin\phi \; Y(t)\right).$$



Phasor diagram of electric field



Variation of X Quadrature with time t.

Coherent State

The number state representation of coherent states

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad ; \; |n\rangle = \text{Fock states}$$

--- (1) $\quad \alpha =$ complex.

This type of coherent state can be produced by displacing the ground (vacuum) state $|0\rangle$.

ie $|\alpha\rangle = D(\alpha)|0\rangle$ --- (2)

Here $D(\alpha) = e^{\alpha a^{\dagger} - \alpha^* a}$ is the displacement operator.

$D(\alpha)$ is a unitary operator hence

$D^{\dagger}(\alpha) D(\alpha) = 1$; $\quad D^{\dagger}(\alpha) = D(-\alpha)$.

$D(\alpha) D^{\dagger}(\alpha) = 1$

Now we can establish the relation (2) as follows -

$$D(\alpha)|0\rangle = e^{\alpha a^{\dagger} - \alpha^* a} |0\rangle$$

$$= e^{\alpha a^{\dagger}} e^{-\alpha^* a} e^{\frac{|\alpha|^2}{2}[a^{\dagger}, a]} |0\rangle$$

$$= e^{\frac{-|\alpha|^2}{2}} e^{\alpha a^{\dagger}} e^{-\alpha^* a} |0\rangle$$

$$= e^{-|\alpha|^2/2} e^{\alpha a^{\dagger}} |0\rangle$$

use BCH formula

$$e^{A+B} = e^{A} e^{B} e^{-\frac{1}{2}[A, B]}$$

provided

$[A, [A, B]] = 0$

$[B, [A, B]] = 0$.

as, $e^{-\alpha^* a}|0\rangle = |0\rangle$

$$\therefore \; D(\alpha)|0\rangle = e^{-|\alpha|^2/2} e^{\alpha a^{\dagger}} |0\rangle$$

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$$D(\alpha)|0\rangle = e^{-|\alpha|^2/2}\, e^{\alpha a^\dagger}|0\rangle$$

$$= e^{-|\alpha|^2/2}\left[1 + \alpha a^\dagger + \frac{\alpha^2}{2!} a^{\dagger 2} + \cdots\right]|0\rangle$$

$$= e^{-|\alpha|^2/2}\left[|0\rangle + \alpha\sqrt{1}\,|1\rangle + \frac{\alpha^2}{2!}\sqrt{2}\,|2\rangle + \cdots\right]$$

$=$ [illegible]

$$D(\alpha)|0\rangle = e^{-|\alpha|^2/2}\left[|0\rangle + \alpha\sqrt{1}\,|1\rangle + \frac{\alpha^2}{2!}\sqrt{2}\cdot\sqrt{1}\,|2\rangle + \cdots\right]$$

$$= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle. = |\alpha\rangle$$

Thus $\boxed{D(\alpha)|0\rangle = |\alpha\rangle}$

Properties of Displacement operator $D(\alpha)$

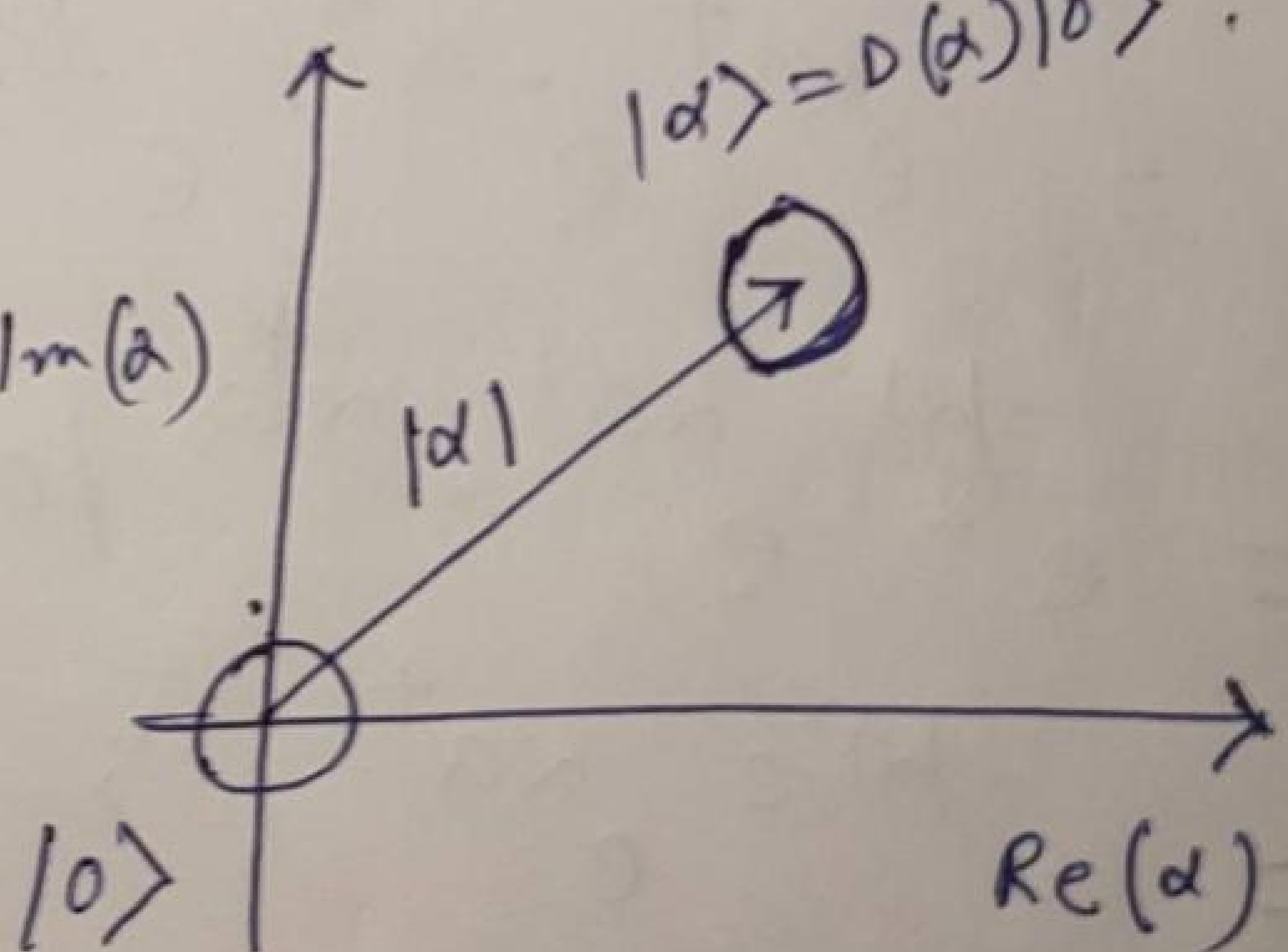
$D(\alpha)$ is a unitary operator

$D^\dagger(\alpha) = D(\alpha)$; $D^\dagger D = D D^\dagger = 1$.

$D(\alpha)|0\rangle = |\alpha\rangle$.

$D^\dagger(\alpha)\, a\, D(\alpha) = a + \alpha$

$D^\dagger(\alpha)\, a^\dagger D(\alpha) = a^\dagger + \alpha^*$



Displacement operator $D(\alpha)$ displaces the vacuum state $|0\rangle$ to coherent state $|\alpha\rangle$

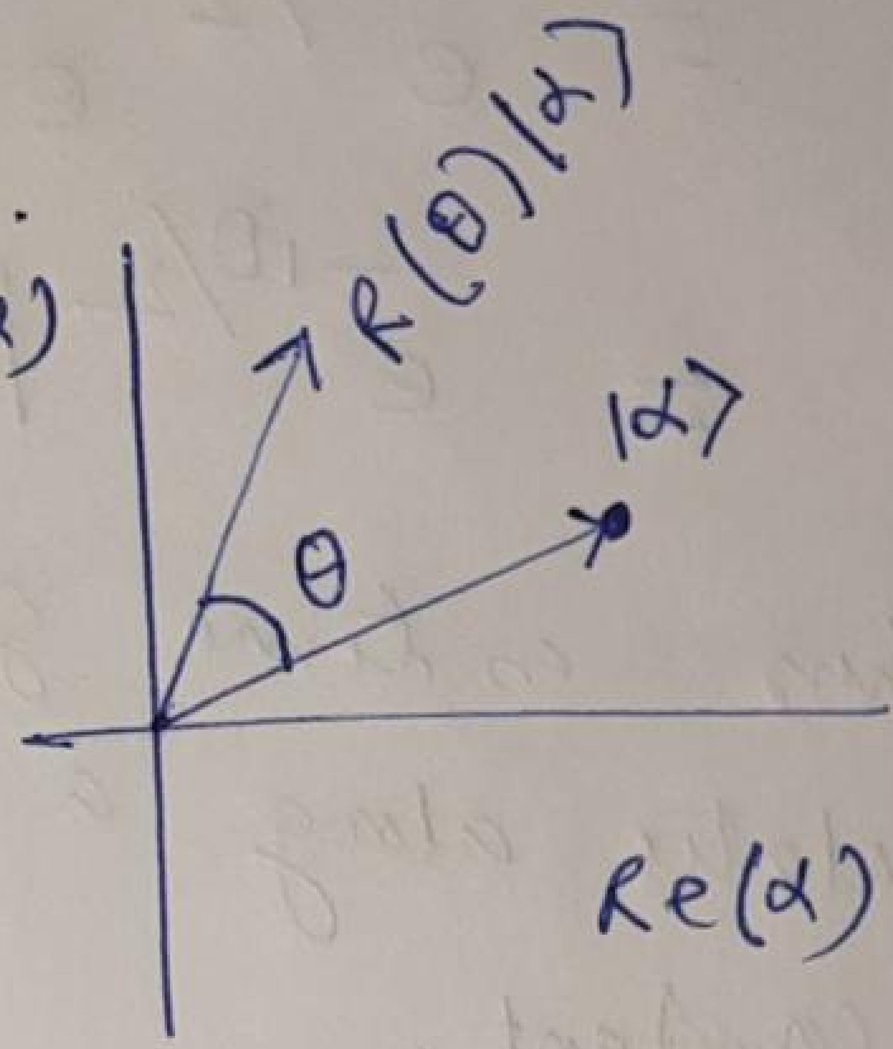
Rotation operator or Phase shifting operator:-

The Rotation operator or phase shifting operator is given by -

$$R(\theta) = e^{-i\theta\hat{N}}$$

Here $\hat{N} = a^{+}a$ is the number operator.

Acting on the coherent state -



$$e^{-i\theta\hat{N}} |\alpha\rangle$$

$$= e^{-i\theta a^{+}a} \, e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-i\theta a^{+}a} |n\rangle.$$

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-i\theta n} |n\rangle$$

$$= e^{-|\alpha e^{-i\theta}|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha e^{-i\theta})^n}{\sqrt{n!}} |n\rangle$$

$$= |\alpha e^{-i\theta}\rangle.$$

Hence $R(\theta)|\alpha\rangle = e^{-i\theta\hat{N}} |\alpha\rangle = |\alpha e^{-i\theta}\rangle$

Thus the Rotation operator $R(\theta)$ Rotates the coherent state by an angle θ.

Time evolution of coherent state $|\alpha\rangle$

Hamiltonian $H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$

Time evolution operator $U = e^{-iHt/\hbar}$

$$U = e^{\frac{-it}{\hbar}\left(a^\dagger a + \frac{1}{2}\right)\hbar\omega} = e^{\frac{-i\omega t}{2}} e^{-i\omega t a^\dagger a}.$$

$$|\alpha_t\rangle = \hat{U}|\alpha_0\rangle$$

$$= e^{-i\theta/2} e^{-i\theta a^\dagger a} |\alpha_0\rangle \quad ; \theta = \omega t.$$

$$= e^{-i\theta/2} |\alpha_0 e^{-i\theta}\rangle. \quad ;$$

Thus as time goes on the coherent state just rotates along a circle keeping it amplitude constant.

Two coherent states $|\alpha\rangle$ and $|\beta\rangle$ are not orthogonal to each other.

$$\langle\alpha|\beta\rangle = e^{-|\alpha-\beta|^2}$$

These two coherent states will be approximately orthogonal. when $|\alpha-\beta| \to \infty$ ie when they are far apart from each other.

Coherent states form a complete set.

$$\int |\alpha\rangle\langle\alpha| \, d^2\alpha = \pi \sum |n\rangle\langle n| = \pi$$

This is the over completeness relation for the coherent states.

The two Quadrature operators do not commute with each other, which can be shown easily.

$$[X,Y] = \frac{1}{4i}\left[a+a^{\dagger}, a-a^{\dagger}\right]$$

$$= \frac{1}{4i}\left[a, -a^{\dagger}\right] + \left[a^{\dagger}, a\right]$$

$$= \frac{1}{4i}(-1-1) = -\frac{2}{4i} = \frac{i}{2}$$

$$[X,Y] = \frac{i}{2}$$

Now we calculate the expectation value of $\hat{X}$ and $\hat{Y}$ Quadrature operator in the coherent state $|\alpha\rangle$.

$$\langle\hat{X}\rangle_\alpha = \langle\alpha|\hat{X}|\alpha\rangle$$

$$= \langle\alpha|\frac{a+a^{\dagger}}{2}|\alpha\rangle = \frac{1}{2}(\alpha+\alpha^*).$$

and $\langle\hat{Y}\rangle_\alpha = \langle\alpha|\hat{Y}|\alpha\rangle$ $\qquad a|\alpha\rangle = \alpha|\alpha\rangle$

$$= \langle\alpha|\frac{a-a^{\dagger}}{2i}|\alpha\rangle \qquad \langle\alpha|a^{\dagger} = \alpha^*\langle\alpha|$$

$$= \frac{1}{2i}(\alpha-\alpha^*)$$

Now $\quad X+iY = \frac{1}{2}(\alpha+\alpha^*) + \frac{1}{2i}(\alpha-\alpha^*)$

$$= \alpha. \quad ; \quad X = \langle\hat{X}\rangle_\alpha, \quad Y = \langle\hat{Y}\rangle_\alpha$$

Comparing with equation –

$$\boxed{X = \mathrm{Re}(\alpha)} \quad \text{and} \quad \boxed{Y = \mathrm{Im}(\alpha)}$$

Now the position operator is given by –

$\hat{x} = x_0 (a + a^{\dagger})$; $x_0 = \sqrt{\frac{\hbar}{2m\omega}}$ is the zero point oscillation –

$\Rightarrow \frac{\hat{x}}{2} = \sqrt{\frac{\hbar}{2m\omega}} \left(\frac{a+a^{\dagger}}{2}\right).$

or $\frac{\hat{x}}{2} = \sqrt{\frac{\hbar}{2m\omega}} \hat{X}$

or $\boxed{\hat{x} = \sqrt{\frac{2\hbar}{m\omega}} \hat{X}}$

Thus multiplying the $\hat{X}$ quadrature operator or dimensionless position operator with appropriate constant $\left(\sqrt{\frac{2\hbar}{m\omega}}\right)$ we can get the position operator. $\hat{x}$.

Taking the average value both sides –

$$\langle \hat{x} \rangle_\alpha = \sqrt{\frac{2\hbar}{m\omega}} \langle \hat{X} \rangle_\alpha .$$

$$\langle \hat{x} \rangle_\alpha = \sqrt{\frac{2\hbar}{m\omega}} Re(\alpha)$$

or $\boxed{Re(\alpha) = \sqrt{\frac{m\omega}{2\hbar}} \langle \hat{x} \rangle_\alpha}$

The expectation value of the photon number operator in coherent state $|\alpha\rangle$

$$\langle n\rangle = \langle\alpha|\hat{n}|\alpha\rangle = \langle\alpha|a^{+}a|\alpha\rangle$$
$$= |\alpha|^2$$

Thus the average number of photon in the coherent state $|\alpha\rangle$ is $|\alpha|^2$

Fluctuation in the photon No. in the coherent state $|\alpha\rangle$.

$$(\Delta n)^2 = \langle \hat{n}^2\rangle_\alpha - \langle\hat{n}\rangle_\alpha^2$$
$$= \langle\alpha|a^{+}aa^{+}a|\alpha\rangle - \left\{\langle\alpha|a^{+}a|\alpha\rangle\right\}^2$$
$$= \alpha^{*}\langle\alpha|aa^{+}|\alpha\rangle\alpha - |\alpha|^4$$
$$= |\alpha|^2\langle\alpha|1+a^{+}a|\alpha\rangle - |\alpha|^4$$
$$= |\alpha|^2\left[1+|\alpha|^2\right] - |\alpha|^4$$
$$= |\alpha|^2 + |\alpha|^4 - |\alpha|^4 = |\alpha|^2$$

$$\Delta n = |\alpha| = \sqrt{\langle n\rangle}.$$

Thus the fluctuation in photon No. in the coherent state $|\alpha\rangle$ is proportional to the average No. of photons in that state.

The probability of finding n photons in the coherent state $|\alpha\rangle$

The probability is given by -

$$P_n = |\langle n|\alpha\rangle|^2$$

Now, $$\langle n|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{\alpha^m}{\sqrt{m!}} \langle n|m\rangle$$

$$= e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{\alpha^m}{\sqrt{m!}} \delta_{nm}$$

$$= e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \delta_{nn} = e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}}$$

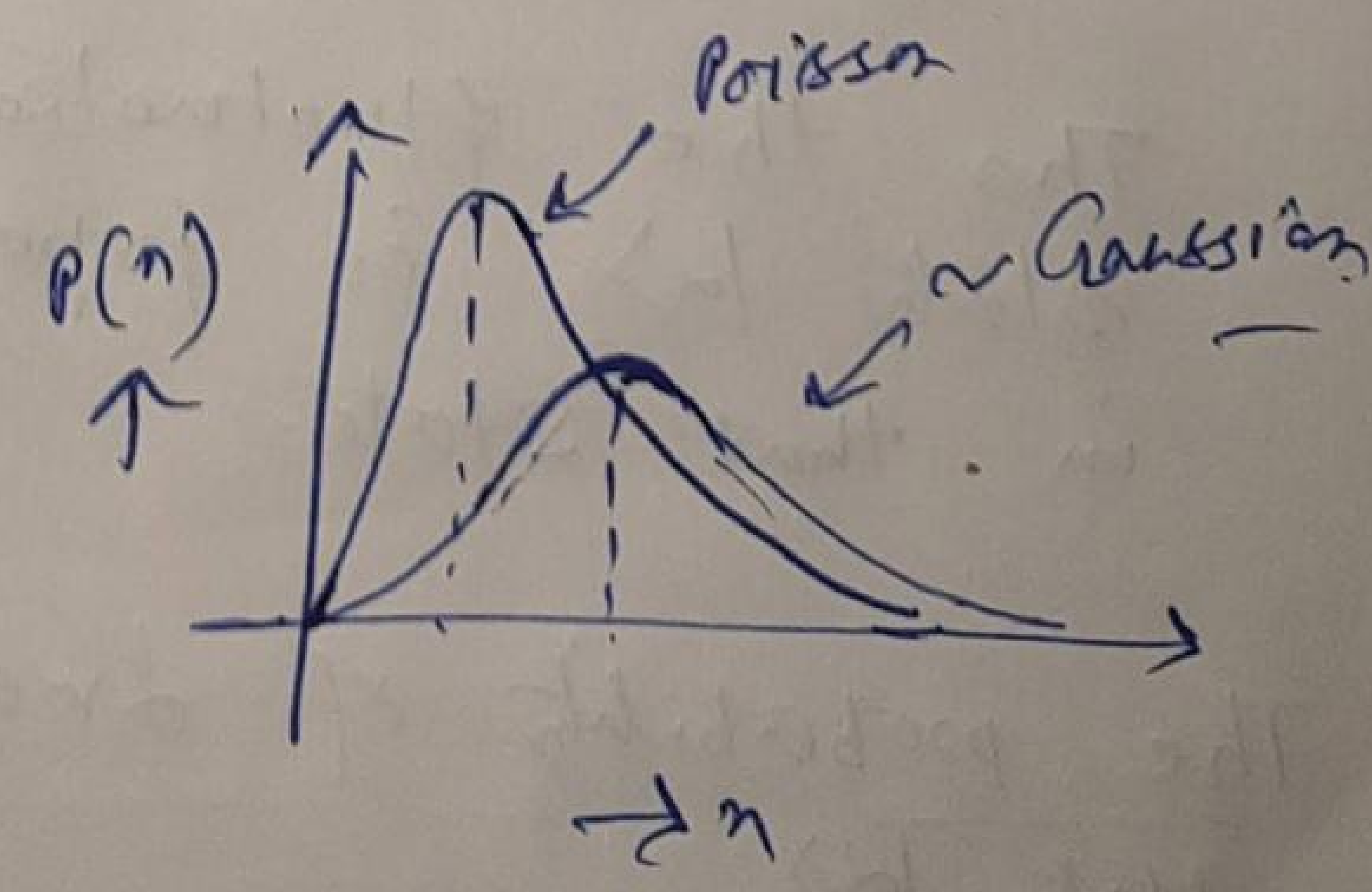
$$P_n = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{\alpha^{2n}}{n!}$$

$$= \frac{e^{-\langle n\rangle} \langle n\rangle^n}{n!}$$

This is the poisson distribution with mean $\langle n\rangle$.

also $\cancel{\frac{\Delta n}{\langle n\rangle}} = \cancel{\frac{1}{\sqrt{\langle n\rangle}}}$

For large n poisson distribution may be approximated as Gaussian.



Position representation of Fock states

The position representation of Number state is given by -

$$\phi_n(x) = \langle x|n\rangle.$$

For $n=0$

$$\phi_0(x) = \langle x|0\rangle$$

This is the ground state wave function.

To find $\phi_0(x)$ we use

$$a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip) \qquad p = -i\hbar\frac{\partial}{\partial x}.$$

$$a|0\rangle = 0.$$

or $$\frac{1}{\sqrt{2\hbar m\omega}}\left(m\omega x - ii\hbar\frac{\partial}{\partial x}\right)\phi_0(x) = 0$$

$$\Rightarrow \left(m\omega x + \hbar\frac{\partial}{\partial x}\right)\phi_0(x) = 0$$

$$\Rightarrow m\omega x\,\phi_0(x) + \hbar\frac{\partial \phi_0}{\partial x} = 0$$

$$\Rightarrow m\omega x\,\phi_0(x) = -\hbar\frac{\partial\phi_0}{\partial x}$$

$$\Rightarrow \int m\omega x\,dx = -\hbar\int\frac{d\phi_0}{\phi_0} + \ln C$$

$$\Rightarrow \frac{m\omega x^2}{2} = -\hbar\ln\phi_0 + \ln C$$

$\left\{ \ln C = \text{Integration Constant} \right.$

$$\Rightarrow \ln\phi_0 = -\frac{m\omega x^2}{2\hbar} + \ln c'$$

$$\ln \phi_0 = -\frac{m\omega x^2}{2\hbar} + \ln c'$$

$$\Rightarrow \phi_0 = K e^{-\frac{m\omega x^2}{2\hbar}} \quad ; \quad K = \ln c'$$

To find K we use the normalization condition

$$\int_{-\infty}^{\infty} \phi_0^*(x)\, \phi_0(x) = 1$$

$$\Rightarrow |N|^2 \int_{-\infty}^{\infty} e^{-\frac{m\omega x^2}{\hbar}} dx = 1$$

$$\Rightarrow N = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}.$$

Thus the ground state wavefunction $\phi_0(x)$ becomes -

$$\phi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

To get ground state wavefunction for the coherent state we just ~~replace~~ displace the position x in positive x direction by some amount say x_0.

$$\phi_\alpha(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}(x-x_0)^2}$$

Both the wavefunction is Gaussian type